Help message for MATLAB function *refval.m*

```
>> help refval
  Function refval determines the geometric and physical constants of the
   equipotential rotational reference ellipsoid of the Geodetic Reference
   System 1980 (GRS80) defined by:
                    a - semi-major axis,
                   GM - geocentric gravitational constant of the Earth,
                   J2 - dynamical form factor and
                omega - angular velocity of the Earth

  For GRS80 the defining parameters are
                    a = 6378137 m
                   GM = 3.986005e+14 m^3/s^2
                   J2 = 1.08263e-03
                omega = 7.292115e-05 rad/s
```

Output from MATLAB function *refval.m*

```
>> refval


GEODETIC REFERENCE SYSTEM 1980
==============================
Defining Constants (exact)

    a =  6378137.000 m          semi-major axis
   GM = 3.986005e+14 m^3/s^2    geocentric gravitational constant
   J2 = 1.082630e-03            dynamical form factor
omega = 7.292115e-05 rad/s      angular velocity

Derived Geometrical Constants of Normal Ellipsoid

    b =  6356752.314140 m       semi-minor axis
    E =   521854.009700 m       linear eccentricity
    c =  6399593.625864 m       polar radius of curvature
   e2 = 6.694380022903e-03      eccentricity squared
  ep2 = 6.739496775482e-03      2nd eccentricity squared
    f = 3.352810681184e-03      flattening
 flat = 298.257222100882        denominator of flattening
    n = 1.679220394629e-03      3rd flattening
    Q = 10001965.729230 m       quadrant distance
   R1 =  6371008.771380 m       mean radius R1 = (2a+b)/3
   R2 =  6371007.180884 m       radius of sphere of same surface area
   R3 =  6371000.789974 m       radius of sphere of same volume

Derived Physical Constants of Normal Ellipsoid

   U0 = 62636860.850046 m^2/s^2 normal gravity potential
   J4 = -2.370912218650e-06     spherical-harmonic coefficient
   J6 = +6.083470628388e-09     spherical-harmonic coefficient
   J8 = -1.426814059713e-11     spherical-harmonic coefficient
    m =  3.449786003078e-03
  g_e =  9.780326771534 m/s^2   normal gravity at equator
  g_p =  9.832186368521 m/s^2   normal gravity at pole
   f* =  0.005302440113         gravity flattening
    k =  0.001931851353         constant in Pizzetti's equation

>>
```

MATLAB function *refval.m*

```
function refval
% Function refval determines the geometric and physical constants of the
%  equipotential rotational reference ellipsoid of the Geodetic Reference
%  System 1980 (GRS80) defined by:
%                  a - semi-major axis,
%                 GM - geocentric gravitational constant of the Earth,
%                 J2 - dynamical form factor and
%              omega - angular velocity of the Earth
%
% For GRS80 the defining parameters are
%                  a = 6378137 m
%                 GM = 3.986005e+14 m^3/s^2
%                 J2 = 1.08263e-03
%              omega = 7.292115e-05 rad/s

%==========================================================================
% Function: refval
%
% Useage:   refval;
%
% Author:
%  Rod Deakin,
%  Department of Mathematical and Geospatial Sciences,
%  RMIT University,
%  GPO Box 2476V, MELBOURNE VIC 3001
%  AUSTRALIA
%  email: rod.deakin@rmit.edu.au
%
% Date:
%  Version 2   21 May 2014
%
%  Note that Version 1 was written as a C++ program in June 1996 with
%  revisions in December 1996 and June 1997.
%
% Functions Required:
%  None
%
% Remarks:
%  Function refval() determines the geometric and physical constants of the
%  equipotential rotational reference ellipsoid defined by:
%              a     - semi-major axis,
%              GM    - geocentric gravitational constant of the Earth,
%              J2    - dynamical form factor and
%              omega - angular velocity of the Earth.
%
%  Given the four defining parameters above, geometric and physical
%  constants can be derived by methods and formulae set out in Ref[1].
%  Theory is given in Ref[2] and Ref[4] and a FORTRAN program listing is
%  given in Ref[3].
%  e2 (first eccentricity squared) is the fundamental derived constant from
%  which the other geometric constants can be computed.  Ref[1] and Ref[4]
%  show how e2 is linked to a, GM, J2 and omega via the formulae
%
%              e2 = 3*J2 + (omega^2)*(a^3)/GM * 4/15*(e^3)/2q0
%
%  This equation must be solved iteratively since e appears on both sides
%  of the equal sign.  A summation formula for 2q0/(e^3) can be derived by
%  considering the equations given in Ref[1] p.398.  Also see ref[4],
%  eq(73), p.28.
%
%  The following geometric constants of the ellipsoid are evaluated
%    b       - semi-minor axis of ellipsoid (metres)
%    c       - polar radius of curvature (metres)
%    ep2     - second eccentricity squared: ep = E/b
%    E       - linear eccentricity: E = sqrt(a^2 - b^2) = a*e
%    f       - flattening of ellipsoid
%    flat    - denominator of flattening: f = 1/flat
%    n       - third flattening of ellipsoid: n = f/(2-f)
%    Q       - quadrant distance of ellipsoid (metres)
%    R1      - radius of sphere having mean radius: R1 = (2*a + b)/3
%    R2      - radius of sphere having same surface area of ellipsoid
%    R3      - radius of sphere having same volume of ellipsoid
%
```

```
%  The following physical constants of the ellipsoid are evaluated
%    gamma_e - normal gravity at equator (m/s^2)
%    gamma_p - normal gravity at pole (m/s^2)
%    J4,J6,J8,... - coefficients of Legendre polynomials in the spherical
%                   harmonic expansion of the gravitational potential V.
%    k       - constant in Pizetti's equation for normal gravity.
%    m       - a derived physical constant: m = omega^2*a^2*b/GM
%    q0      - q(subscript zero), physical constant used in computation of
%              gamma_e, gamma_p
%    qp0     - q-primed(subscript zero), physical constant used in
%              computation of gamma_e, gamma_p
%    U0      - normal gravity potential at ellipsoid (m^2/s^2)
%
% Variables:
%  a       - semi-major axis of ellipsoid (metres)
%  b       - semi-minor axis of ellipsoid (metres)
%  c       - polar radius of curvature (metres)
%  c0      - coefficient in computation of quadrant distance Q
%  e       - (first) eccentricity of ellipsoid: e = E/a
%  e2      - (first) eccentricity squared
%  e21     - approximate value of e2
%  ep      - second eccentricity: ep = E/b
%  ep2     - second eccentricity squared
%  E       - linear eccentricity: E = sqrt(a^2 - b^2) = a*e
%  f       - flattening of ellipsoid: f = (a-b)/a = 1-sqrt(1-e^2)
%  f1      - gravity flattening
%  flat    - denominator of flattening: f = 1/flat
%  gamma_e - normal gravity at equator (m/s^2)
%  gamma_p - normal gravity at pole (m/s^2)
%  GM      - geocentric gravitational constant (m^3/s^2)
%  i,j     - integer counters
%  Jvec    - vector of zonal harmonic terms for computation of normal
%            gravitational potential V
%  J2      - dynamical form factor
%  k       - constant in Pizetti's equation for normal gravity.
%  m       - a derived physical constant: m = omega^2*a^2*b/GM = m1/a*b;
%  m1      - a derived physical constant: m1 = omega^2*a^3/GM = m/b*a;
%  n       - third flattening of ellipsoid: n = f/(2-f)
%  n2,n4,... even powers of n
%  N       - size of vector Jvec
%  omega   - angular velocity of earth (radians/sec)
%  q0      - q(subscript zero), physical constant used in computation of
%            gamma_e, gamma_p
%  qp0     - q-primed(subscript zero), physical constant used in
%            computation of gamma_e, gamma_p
%  Q       - quadrant distance of ellipsoid (metres)
%  R1      - radius of sphere having mean radius: R1 = (2*a + b)/3
%  R2      - radius of sphere having same surface area as ellipsoid
%  R3      - radius of sphere having same volume as ellipsoid
%  sgn     - sgn = 1 or sgn = -1
%  U0      - normal gravity potential at ellipsoid (m^2/s^2)
%  x       - local variable
%
% References:
%  1. Moritz, H., 1980, 'Geodetic Reference System 1980',  Bulletin
%        Geodesique Vol.54(3), 1980, pp.395-405.
%  2. Heiskanen, W.A. and Moritz, H. (1967).  Physical Geodesy, W.H.Freeman
%        and Co., London, 364 pages.
%  3. Tscherning, C.C., Rapp, R.H. and Goad C., (1983).  'A Comparison of
%        methods for Computing Gravimetric Quantities from High Degree
%        Spherical Harmonic Expansions',  Manuscripta Geodaetica, Vol.8,
%        1983, p.249-272.
%  4. Deakin, R.E., 1997. 'The Normal Gravity Field', Private Notes,
%        Department of Geospatial Science, RMIT University, 38 pages.
%  5. Deakin, R.E. & Hunter, M.N., 2012. 'A Fresh Look at the UTM
%        Projection: Karney-Krueger equations', Presented at the Surveying
%        and Spatial Sciences Institute (SSSI) Land Surveying Commission
%        National Conference,  Melbourne, 18-21 April, 2012.
%  6. Deakin, R.E. & Hunter, M.N., 2013. 'Geometric Geodesy Part A', School
%        of Mathematical & Geospatial Sciences, RMIT University, 3rd
%        printing, January 2013.
%============================================================================
```

```
%------------------------------------------------------------------------
% set defining constants of rotational equipotential ellipsoid for GRS80
%------------------------------------------------------------------------
a = 6378137.0;
GM = 3.986005e+14;
J2 = 1.08263e-03;
omega = 7.292115e-05;

%---------------------------------------
% compute e2 (first eccentricity squared)
%---------------------------------------
% See Ref[1], p.398
% set initial value of first eccentricity squared
m1  = omega^2*a^3/GM;
e21 = 3*J2 + m1;
% set e2 = initial value
e2  = e21;
for i = 1:10
    ep2   = e2/(1-e2);
    x     = (1/(1-e2))^(3/2);
    sgn   = 1;
    twoq0 = 0;
    % this loop calculates 2q0/(e^3)
    for j = 1:20
        twoq0 = twoq0 + (sgn*(4*j/(2*j+1)/(2*j+3)*x));
        sgn   = -sgn;
        x     = x*ep2;
    end
    % set e2 = initial value + correction
    e2 = e21 + (m1*((4/15/twoq0)-1));
end
% e2 now known, calculate geometric constants of ellipsoid

%-------------------------------------------------------
% compute GEOMETRIC constants of equipotential ellipsoid
%-------------------------------------------------------
e    = sqrt(e2);              % eccentricity of ellipsoid
ep2  = e2/(1-e2);             % 2nd eccentricity squared
ep   = sqrt(ep2);             % 2nd eccentricity
f    = 1 - sqrt(1-e2);        % flattening
flat = 1/f;                   % denominator of flattening
b    = a*(1-f);               % semi-minor axis of ellipsoid
E    = a*e;                   % linear eccentricity
c    = a*a/b;                 % polar radius of curvature
n    = f/(2-f);               % 3rd flattening

% computation of quadrant length of ellipsoid.  See Ref[5], eq (39).
n2 = n*n;      % even powers of n
n4 = n2*n2;
n6 = n4*n2;
n8 = n6*n2;
c0 = 1 + 1/4*n2 + 1/64*n4 + 1/256*n6 + 25/16384*n8;
Q = a/(1+n)*c0*(pi/2);       % quadrant distance

% computation of radii of equivalent spheres.  See Ref[6], pp. 86-87.
R1 = (2*a+b)/3;
R2 = sqrt(a^2/2*(1+(1-e2)/(2*e)*log((1+e)/(1-e))));
R3 = (a^2*b)^(1/3);

%------------------------------------------------------
% compute PHYSICAL constants of equipotential ellipsoid
%------------------------------------------------------

% compute normal potential of the reference ellipsoid U0
% See Ref[1] pp.399-400, Ref[2] p.67, eq.(2-61), Ref[4] eq.(38b)
U0 = GM/E*atan(ep) + omega^2*a^2/3;

% compute normal gravity at equator and poles
% formula for q0 given in Ref[2] eq.(2-58), p.66 and Ref[4] eq.(31), p.12
% see also Ref[1] p.398
q0 = ((1+(3/ep2))*atan(ep)-(3/ep))/2;
% formula for qp0 given in Ref[2] eq.(2-67), p.68 and Ref[4] eq.(52), p.19
% see also Ref[1] p.400; Ref[4] eq.(52)
qp0 = 3*(1+(1/ep2))*(1-(1/ep*atan(ep))) - 1;
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```
% compute normal gravity at the equator and the pole
% formulae for gamma_e and gamma_p given in Ref[2] eqs (2-73) and (2-74),
% p.69; Ref[1] p.400 and Ref[4], eqs (62a),(62b).
m       = m1/a*b;
gamma_e = GM/a/b*(1-m-(m/6*ep*qp0/q0));
gamma_p = GM/a/a*(1+(m/3*ep*qp0/q0));
% compute gravity flattening
f1 = gamma_p/gamma_e - 1;
% compute constant k in Pizetti's equation for normal gravity
% formula for k in Ref[1], p.401 and Ref[4] eq.(68), p.26.
k = b*gamma_p/(a*gamma_e) - 1;

%---compute the coefficients J4, J6, J8, ... J20
% set dimensions of vector Jvec()
N = 20;
Jvec = zeros(N,1);
% This vector is used store the coeff's J2, J4, J6, ... J20 where J(2n) are
% related to J2.  See Ref[2], p.73, eqs. (2-92),(2-92') and Ref[4] eq.(86)
% Also Jvec(1,1) = Jvec(3,1) = Jvec(5,1) = ... = Jvec(odd,1) = 0.
% The elements of Jvec() are used in computations of the normal potential V
% and its derivatives.  Note also that the elements of Jvec() are zonal
% terms, i.e. the order is zero (m=0) thus they are also equal to quasi-
% normalised coefficients.  See Ref[3], eq. 21, p. 258.

% set value of Jvec(2,1)
Jvec(2,1) = J2;
% set even values of Jvec()   */
for j=2:N/2
   sgn = (-1)^(j+1);
   x1 = 3/(2*j+1)/(2*j+3)*e2^j;
   x2 = 1 - j + (5*j*J2/e2);
   Jvec(2*j,1) = sgn * x1 * x2;
end

% print headings and computed data
fprintf('\n\nGEODETIC REFERENCE SYSTEM 1980');
fprintf('\n==============================');
fprintf('\nDefining Constants (exact) \n');
fprintf('\n    a = %12.3f m          semi-major axis',a);
fprintf('\n   GM = %9.6e m^3/s^2    geocentric gravitational constant',GM);
fprintf('\n   J2 = %9.6e            dynamical form factor',J2);
fprintf('\nomega = %9.6e rad/s      angular velocity',omega);

fprintf('\n\nDerived Geometrical Constants of Normal Ellipsoid \n');
fprintf('\n    b = %15.6f m       semi-minor axis',b);
fprintf('\n    E = %15.6f m       linear eccentricity',E);
fprintf('\n    c = %15.6f m       polar radius of curvature',c);
fprintf('\n   e2 = %15.12e      eccentricity squared',e2);
fprintf('\n  ep2 = %15.12e      2nd eccentricity squared',ep2);
fprintf('\n    f = %15.12e      flattening',f);
fprintf('\n flat = %15.12f        denominator of flattening',flat);
fprintf('\n    n = %15.12e      3rd flattening',n);
fprintf('\n    Q = %15.6f m       quadrant distance',Q);
fprintf('\n   R1 = %15.6f m       mean radius R1 = (2a+b)/3',R1);
fprintf('\n   R2 = %15.6f m       radius of sphere of same surface area',R2);
fprintf('\n   R3 = %15.6f m       radius of sphere of same volume',R3);

fprintf('\n\nDerived Physical Constants of Normal Ellipsoid \n');
fprintf('\n   U0 = %15.6f m^2/s^2 normal gravity potential',U0);
fprintf('\n   J4 = %+15.12e     spherical-harmonic coefficient',Jvec(4));
fprintf('\n   J6 = %+15.12e     spherical-harmonic coefficient',Jvec(6));
fprintf('\n   J8 = %+15.12e     spherical-harmonic coefficient',Jvec(8));
fprintf('\n    m = % 15.12e',m);
fprintf('\n  g_e = %15.12f m/s^2   normal gravity at equator',gamma_e);
fprintf('\n  g_p = %15.12f m/s^2   normal gravity at pole',gamma_p);
fprintf('\n   f* = %15.12f         gravity flattening',f1);
fprintf('\n    k = %15.12f         constant in Pizzetti''s equation',k);

fprintf('\n\n');
```